Lake contamination models…

"*The time to reach steady state in a perfectly mixed reservoir can be derived from the time that it takes for the term* exp[-t/R] *go to ≈ zero, which occurs if* t = 6R, *when 99.75% of* C_{ssp} *has been reached (600 months in the case of the model lake).*" J.C. Varekamp. 2003. Lake contamination models for evolution towards steady state. *J. Limnol.*, 62(Suppl.1): 67-72.

The above sentence deserves critical consideration on the grounds of physical and experimental arguments. In an elementary physical system where a capacitor (*C* farad) is fed a constant electromotive force (volt) with some resistance (*R* ohm), the electrical charge (*q* coulomb) varies over time as $q = q_{max} (1-e^{-t/RC})$. Using this equation, we can determine the time necessary for the charge to attain some arbitrary fraction of its final value, say *0.9 q_{max}* or *0.999 q_{max}*. This choice is somewhat arbitrary and we must constrain it based on physical considerations. The magnitudes of the experimental uncertainties involved can help define the problem in a coherent way. Let us suppose that, in the capacitor example, charge can be measured within ±5% and *q_{max} = 100 microC*. All the measurements performed near the steady state will range in the interval 95-105 microC. This means that, with a very conservative hypothesis, our capacitive constant *RC* is compatible with experimental constraints, and will be defined by the ratio $q/q_{max} \approx 0.95$. It is not physically tenable to have $q/q_{max} > 0.95$.

Returning to Varekamp's paper, on page 70 we read: "*Precision of the chemical analysis is ±4%...*". From our previous analysis we adopt $C_p /C^{ss}_p = 0.96$ in order to find a physically tenable time that is compatible with the experimentally defined steady state. We get $t/R_p = 3.22$ instead of the $t/R_p = 6$ reported by Varekamp.

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J.C. Varekamp reply to comment

Varekamp (2003) stated that it will take about 6 residence times to reach a state where the total amount or concentration of a substance in a reservoir is 99.75% of the steady state amount (or concentration). There is nothing wrong with that statement *per se*, it simply follows from the expressions of evolution towards steady state. When we want to invert such theoretical predictive statements, and e.g., want to determine how long a perturbance from an earlier steady state has been occurring in a natural system, we have to worry about the data precision. The example provided and the analogy with the charging of a capacitor in the comment

by Guala-Valverde point this out quite clearly, and I fully agree. The situation in the real world carries actually still greater uncertainties than those expressed by Guala-Valverde, and it may be useful to explore how measurement errors propagate in these systems. The analytical data from a reservoir such as a lake have their uncertainty, but also the steady state concentration in the reservoir has an inherent uncertainty.

The steady state concentration is derived from the mass influx of the substance of interest into the reservoir and the residence time (variations of expression 5 in Varekamp, 2003). The mass influx term is usually determined from river water flux measurements and water analyses, and may have up to +10% relative error, which may translate into an up to +10% error in the calculated steady state concentration. In the simple case of a lake with one inlet and one outlet, errors in the water influx measurement do not propagate into the calculated steady state concentration, because both the water balance (residence time) and pollutant balance are equally impacted.

When the water balance is largely determined by several large inlets and the pollutant balance by a smaller inlet, the residence time is not very sensitive to measurement errors in the influx with the pollutant, and in such cases, the error in the flux measurements propagates into the calculated steady state concentration. The assessment of proximity to steady state or time elapsed since a change in pollutant fluxes has to be evaluated taking into account the uncertainty in steady state concentration and the uncertainty in the measurement of the lake water composition. As an extreme example, evolution curves are drawn for a reservoir that has a steady state concentration of 100 concentration units ±10% (Fig. 1).
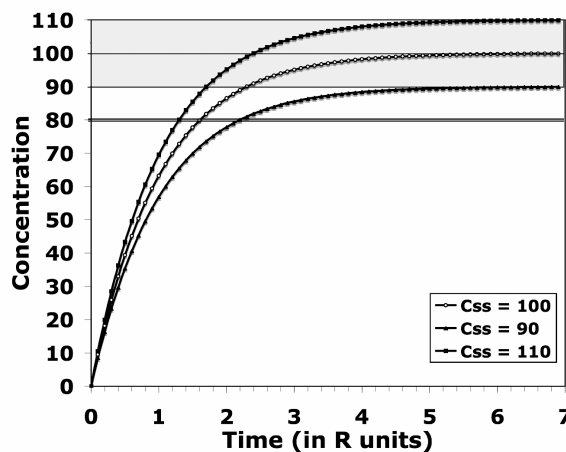

Fig. 1. Concentration-time evolution curves for a lake that may reach, 90, 100 or 110 concentration units. A measured water concentration of 80 units would give a range of possible times elapsed since the system started to evolve.
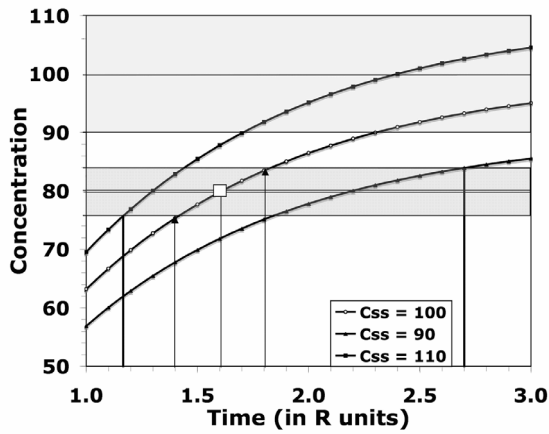
Fig. 2. Detail of figure 1 with the three evolution curves, and the window of uncertainty for the measured data (80 ± 5%). The range of elapsed times taking all errors into account are indicated by the two heavy vertical lines (1.15 to 2.7 residence times); the range of times related to the analytical error in the lake water only is shown with the two arrows (1.4 to 1.8 residence times). Without error analysis, the result would be 1.6 residence times (open square).

If we assume a ±5% error in the analytical data, the estimates for the time elapsed in this system for a measured 80 concentration units range from 1.15 to 2.7 residence times when taking all errors into account, and from 1.4 to 1.8 residence times for the analytical error in the lake water only (Fig. 2).

When a system is closer to steady state, the errors will become much larger because of the flat slope of the evolution curves, whereas the errors will be very small for systems far removed from steady state. So although much of my paper presented theoretical or predictive views, one has to be careful applying these equations for reconstructions of the past from real data (as done in the second part of my paper), and should carefully evaluate the potential errors involved. Thanks to Dr. Guala-Valverde for pointing this out.

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