JOURNAL OF LIMNOLOGY

DOI: 10.4081/jlimnol.2019.1813

SUPPLEMENTARY MATERIAL

A new technology for suspended sediment simulation in Lake Taihu, China: Combination of hydrodynamic modeling and remote sensing

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Appendix A

Details of the horizontal diffusion F_x , F_y and F_s is given as follows

$$F_x = \frac{\partial}{\partial x} \left[2A_m h \frac{\partial u}{\partial x} \right] + \frac{\partial}{\partial y} \left[A_m h \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) \right]$$
(A1)

$$F_{y} = \frac{\partial}{\partial x} \left[A_{m} h \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) \right] + \frac{\partial}{\partial y} \left[2A_{m} h \frac{\partial u}{\partial y} \right]$$
(A2)

$$F_{s} = \frac{\partial}{\partial x} \left[A_{m} h \frac{\partial S}{\partial y} \right] + \frac{\partial}{\partial y} \left[A_{m} h \frac{\partial S}{\partial y} \right]$$
(A3)

where A_m is the horizontal viscosity coefficient.

Appendix B

Test of the sediment model

This case tests the primary dynamics of suspended sediment model including treatment of settling, erosion, vertical diffusion and advection. A classic problem about suspended sediment is the vertical distribution of suspended sediment at equilibrium state in a flume, which can be depicted by Rouse (1938) distribution. If the diffusion coefficient is assumed to be a constant, then Rouse distribution is written as

$$S = S_a \exp\left[\frac{\omega_s}{\upsilon_s}(z-a)\right]$$
(B1)

where S_a is the sediment concentration at the referenced depth z = a.

For the parabolic diffusion coefficient, Rouse distribution is written as

$$S = S_a \left[\frac{a(H-z)}{z(H-a)} \right]^{\frac{\omega}{\kappa u_*}}$$
(B2)

where k = 0.4 is the von Kármán constant, and u_* is the friction velocity. More details about the derivation of the above equations are provided in van Rijn's book (1993).

To verify the criterion in dealing with erosion and deposition in the suspended sediment, a



numerical water channel was set up (Fig. S1). The inflow was imposed on the left boundary, and the fixed water level was imposed on the right boundary. The erodible bottom was set on the bed. The bed load can be eroded and suspended into the water. After the model ramps up to an equilibrium state, the vertical distribution of the sediment should be close to a Rouse profile. Some parameters about the water channel are shown in Tab. B1. The horizontal distribution of the sediment concentration along the channel at the height of 0.5 m above the bed is shown in Fig. S2. The comparison between the model results under the time steps of 30 s (Exp. 1) and 60 s (Exp. 2) and the analytic solutions were carried out. The parameters were given by $\omega = 0.001 \text{ m/s}$, H = 10 m, $u_* = 0.017 \text{ m/s}$, a = 0.5 m. Fig. S3 demonstrates that the agreement between the model results and the analytic solution is well even though some errors were introduced in the numerical model.



Fig. S1. The grid of the water channel.



Fig. S2. The sediment concentration at 0.5 m height above the bottom, along the channel.





Fig. S3. Comparison of the numerical results and the analytic solution of the sediment distribution under (a) Constant diffusivity; (b) Parabolic diffusivity. Exp. 1 denotes the numerical results with a time step of 60 s and Exp. 2 denotes that of 30 s.



Grid nodes	1280
Grid elements	2338
Vertical layers	10
Channel length	5000 m
Channel width	500 m
Channel height	10 m
Maximum resolution	50 m

Tab. B1. The parameters of the numerical water channel.

Appendix C

Derivation of the equation

Depth-integrating the eq. (5) yields:

$$\frac{\partial}{\partial t} \int_{-h}^{\eta} Sdz - S(\eta) \frac{\partial \eta}{\partial t} + \frac{\partial}{\partial x} \int_{-h}^{\eta} uSdz - u(\eta)S(\eta) \frac{\partial \eta}{\partial x} - u(-h)S(-h) \frac{\partial h}{\partial x} + \frac{\partial}{\partial y} \int_{-h}^{\eta} vSdz - v(\eta)S(\eta) \frac{\partial \eta}{\partial y} - v(-h)S(-h) \frac{\partial h}{\partial y} + [w(\eta) - \omega_s]S(\eta) - [w(-h) - \omega_s]S(-h) = \upsilon_s \frac{\partial S}{\partial z} \Big|_{\eta} - \upsilon_s \frac{\partial S}{\partial z} \Big|_{-h}$$
(C1)

Using the surface and bottom boundary conditions:

$$w(\eta) = \frac{\partial \eta}{\partial t} + u(\eta)\frac{\partial \eta}{\partial x} + v(\eta)\frac{\partial \eta}{\partial y}$$
(C2)

$$w(-h) = -u(-h)\frac{\partial h}{\partial x} - v(-h)\frac{\partial h}{\partial y}$$
(C3)

$$\left(\upsilon_s \frac{\partial S}{\partial z} + S\omega_s\right)\Big|_{\eta} = 0 \tag{C4}$$

Then eq. (C1) becomes:

$$\frac{\partial}{\partial t} \int_{-h}^{\eta} S dz + \frac{\partial}{\partial x} \int_{-h}^{\eta} u S dz + \frac{\partial}{\partial y} \int_{-h}^{\eta} v S dz = -\omega_s S(-h) - \upsilon_s \frac{\partial S}{\partial z} \Big|_{-h}$$
(C5)



The right-hand terms are deposition and erosion flux respectively, expressed as:

$$\frac{\partial}{\partial t} \int_{-h}^{\eta} Sdz + \frac{\partial}{\partial x} \int_{-h}^{\eta} uSdz + \frac{\partial}{\partial y} \int_{-h}^{\eta} vSdz = -D + E$$
(C6)

where $E = -\upsilon_s \frac{\partial S}{\partial z}|_{-h}$ and $D = \omega_s S(-h)$

Integrating Eq. (16) over the period from t_1 to t_2 we obtain:

$$\int_{t_1}^{t_2} Edt = \int_{-h}^{\eta_2} S(t_2) dz - \int_{-h}^{\eta_1} S(t_1) dz + \int_{t_1}^{t_2} A dv dt + \int_{t_1}^{t_2} D dt$$
(C7)

where $F_{Adv} = \frac{\partial}{\partial x} \int_{-h}^{\eta} uSdz + \frac{\partial}{\partial y} \int_{-h}^{\eta} vSdz$ denotes the advection transport.

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